Electronic Journal of Differential Equations, 2018, vol.2018

# Fundamental solutions of two multidimensional elliptic equations

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## Abstract

 We construct fundamental solutions for two-multidimensional elliptic equations. The solutions are written in explicit form via hypergeometric Gauss functions for λ = 0 and via confluent Horn functions for λ 6≠ 0. It is proved that the fundamental solutions found possess power-type singularity ρ2-nas ρ → 0.

## Keywords

Confluent Horn function, Fundamental solution, Hypergeometric Gauss function, Transmutation operator

## References

[1] G. E. Andrews, R. Askey, R. Roy; Special functions, Cambridge Univ. Press., Cambridge, 1999.

[2] K.I. Babenko; On the theory of mixed-type equations, Doctoral dissertation in mathematics and physics, 1952.

[3] R. Bader, P. Germain; Sur quelques problms relatifs a lquation du type mixte de Tricomi, O.N.E.R.A. Publ., 54 (1952), II+57 pp.

[4] A. V. Bitsadze; Some classes of partial differential equations, [in Russian]: Nauka, Moscow, 1981; English transl.: Gordon & Breach, New York, 1988.

[5] S. A. Chaplygin; On gas-like structures, Moscow-Leningrad: Gostekhizdat, 1949.

[6] H. S. Cohl, J. T. Conway; Exact Fourier expansion in cylindrical coordinates for the threedimensional Helmholtz Green function, Z. Anges. Math. Phys., 61 (2010) pp. 425-443.

[7] A. Erdelyi, W. Magnus, F. Oberhettinger, F.G. Tricomi; Higher transcendental functions, Vol. I New York, Toronto and London: McGraw-Hill Book Company, 1953.

[8] F. Frankl; Sulla teoria dellequazione yzxx +zyy = 0, Izvestia Akad. Nauk S.S.S.R. Ser. Math., 10 (1946), pp. 135-136.

[9] F. Frankl; Sul problema di Chaplygin per flusso misto subsonico e supersonico, Izvestia Akad. Nauk S.S.S.R. Ser. Math., 9 (1945), pp. 121-143.

[10] F. Frankl; Sul problema di Cauchy per equazioni a derivate parziali di tipo misto ellitticoiperbolico con dati iniziali sulla linea parabolica, Izvestia Akad. Nauk S.S.S.R. Ser. Math., 8 (1944), pp. 195-224.

[11] M. Frigon, R. L. Pouso; Theory and applications of first-order systems of Stieltjes differential equations, Advances in Nonlinear Analysis 6 (2017), no. 1, pp. 13-36.

[12] I. B. Garipov, R. M. Mavlyaviev; Fundamental solution of multidimensional axisymmetric Helmholtz equation, Complex Variables and Elliptic Equations, 62 (2017), pp. 287-296.

[13] S. Gellerstedt; Sur une equation lineare aux derivees partielles de type mixte, Arkiv. Mat., Ast. och Fysik. 25 A (1937), no. 29.

[14] S. Gellerstedt; Sur un probleme aux limites pour lequation y 2szxx + zyy = 0, Arkiv Mat., Ast. och Fysik. 25 A (1935), no. 10.

[15] S. Gellerstedt; Sur un probleme aux limites pour une equation lineaire aux derivees partielles du second orde de tipe mixte, These, Uppsala, 1935.

[16] M. Ghergu, V. Radulescu; Nonlinear PDEs. Mathematical Models in Biology, Chemistry and Population Genetics, Springer Monographs in Mathematics. Springer, Heidelberg, 2012.

[17] A. Hasanov, M.S. Salakhitdinov; The fundamental solution for one class of degenerate elliptic equations, More Progresses in Analysis: Proceedings of the 5th International ISAAC Congress, Catania, Italy, 25 -30 July 2005 (H. G. W. Begehr, F. Nicolosi), (2005), pp. 521-531

[18] A. Hasanov, J.M. Rassias; Fundamental Solutions of two degenerated elliptic equations and solutions of boundary value problems in infinite area, International Journal of Applied Mathematics & Statistics, 8, M07 (2007), pp. 87-95.

[19] A. Hasanov, R. Seilkhanova; Particular solutions of generalized Euler-Poisson-Darboux equation, Electron. J. Diff. Equ., Vol. 2015 (2015), no. 09, pp. 1-10.

[20] E. Holmgren; Sur un probleme aux limites pour l'equation ymzxx + zyy = 0, Arkiv Mat., Astr., och Fysik. 19 B (1926), no. 14.

[21] M. B. Kapilevich; On confluent Horn functions, Differentsial'nye uravneniya, 2 (1966) no. 9, pp. 1239-1254.

[22] P. W. Karlsson, H.M. Srivastava; Multiple Gaussian gypergeometric series, Ellis Horwood, 1985.

[23] M. V. Keldysh; On certain cases of degeneration of equations of elliptic type on the boundry of a domain, Doklady Akademii Nauk SSSR, 77 (1951), pp. 181-183.

[24] B. M. Levitan; Bessel function expansions in series and Fourier integrals, Uspekhi Mat. Nauk, 6 (1951), pp. 102-143.

[25] A. M. Nigmedzyanova; Integral representation of solution to a multidimensional degenerating elliptic equation of 1st kind with positive parameter, Izvestiya Tulskoho gosudarstvennogo universiteta. Estesstvennye nauki, 3 (2014), pp. 19-33.

[26] A. M. Nigmedzyanova; On fundamental solution of one degenerate elliptic equation, in Proceedings of Second All-Russia Scientific Conference "Mathematical Modeling and BoundaryValue Problems". Part 3, Samara, 2005 (Samara State University, Samara, 2005), pp. 180-182.

[27] M. Tanda; Alien derivatives of the WKB solutions of the Gauss hypergeometric differential equation with a large parameter, Opuscula Math. 35 (2015), no. 5, pp. 803-823.

[28] F. Tricomi; Ancora sull'equazione yzxx + zyy = 0, Rend. Acc. Lincei, Ser. 6, 6 (1927), pp. 567-571.

[29] F. Tricomi; Ulteriori richerche sull'equazione yzxx + zyy = 0, Rendiconti del Circolo Matematico di Palermo, 52 (1928), pp. 63-90.

[30] I. N. Vekua; Sur une generalisation de l'integrale de Poisson pour le demi-plan, C. R. Acad. Sci. URSS, 56 (1947) no. 3, pp. 229-231.

[31] V. F. Volkodavov; Solution of the N problem for the general Tricomi equation, Proceedings of the first scientific conference of mathematical departments of the pedagogical institutes of the Volga Region, Kujbyshev skog, (1961), pp. 49-52.